The Gravitational Field

The use of multimedia in teaching physics

Texts to multimedia presentation

Jan Hrnčíř
jan.hrncir@gfxs.cz

Martin Klejch
martin.klejch@gfxs.cz

F. X. Šalda Grammar School, Liberec
Czech Republic

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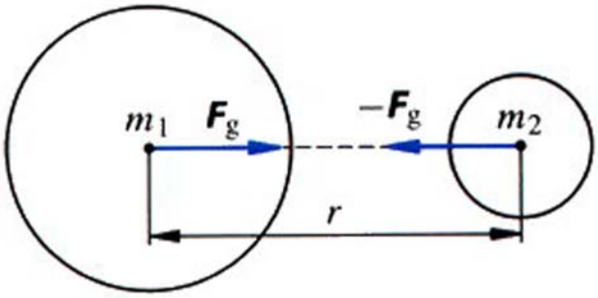
1. Gravitational Field

You will get to know Newton's laws and you will understand the basic patterns, which affect the bodies' movements in a gravitational field in this chapter. You should understand why they happen actually. You can also find there some physical problems, so you can test your new knowledge.

1. 1. Newton's law of gravitation

Isaac Newton studied the gravitational forces' behaviour in the 17^{th} century, pursuant to the observation of movements of the Moon around the Earth and the planets around the Sun. A revolutionary idea hit him: The gravitational force causes it all! The formula, which he deduced for the gravitational forces' value, is called Newton's law of gravitation.



Every two homogeneous bodies with spherical shape gravitate to each other by the same valued gravitational forces $\boldsymbol{F_g}$ and $-\boldsymbol{F_g}$, which have opposite directions.

$$F_g = \kappa \cdot \frac{m_1 \cdot m_2}{r^2}$$

κ is the gravitational constant

$\kappa \doteq 6{,}67 \cdot 10^{-11}\ \mathrm{N \cdot m^2 \cdot kg^{-2}}$

The gravitational constant is now specified as *G*.

We can use this formula also for bodies, which are not homogeneous and also for bodies with other shapes than a sphere, if their proportions could be vanished in regard of their distance, so we can consider them as mass points (for example Earth – Moon, Sun – comet, ISS (space station) – Earth). The bodies on the Earth's surface are gravitated onto it according to this gravitational law (*r* is the Earth's radius).

1. 2. Intensity of the gravitational field

A gravitational field exists around every body and it affects other bodies. The intensity of a gravitational field was created for comparison of the force's impacts in different places of a gravitational field.

We define the intensity of a gravitational field ***K*** in given place as a quotient of the gravitational force $\boldsymbol{F_g}$, which affects the mass point in the place and the point's weight *m*.

$$\mathbf{K} = \frac{\mathbf{F_g}}{\mathbf{m}}$$

The gravitational field's intensity ***K*** is a vector magnitude with the same direction as the gravitational force $\boldsymbol{F_g}$, which affects the mass point in the given place. [K] = $N.kg^{-1}$.

We can get the value of the gravitational field's intensity from the formula for the gravitational force's value from the gravitational law.

$$F_g = \kappa \cdot \frac{m \cdot M}{r^2} \qquad K = \kappa \cdot \frac{M}{r^2}$$

1. 3. Central gravitational field

The vector of the gravitational field's intensity leads to the centre of the body of weight *M*. That field is a central gravitational field and the body's centre is the centre of the central gravitational field. The value of the gravitational field's intensity in height *h* over the Earth's surface is:

$$K = \kappa \cdot \frac{M_Z}{(R_Z + h)^2}$$

M_Z is the Earth's weight ($5{,}98 \cdot 10^{24}$ kg), R_Z is the Earth's radius ($6{,}37 \cdot 10^{6}$ m). The value of the gravitational field's intensity decreases with increasing height over the Earth's surface.

1. 4. Homogenous gravitational field

If we pursue the gravitational field on small areas, for example on an area of one hundred square metres, the gravitational field can be considered as a homogenous one. The intensity in a homogenous gravitational field is constant.

2. Gravitational and gravity acceleration

You will get to know the basic patterns, which affect the Earth's gravity and you will understand the difference between the gravity and gravitational forces in this chapter. There is also a physical problem for you, so you can test your knowledge.

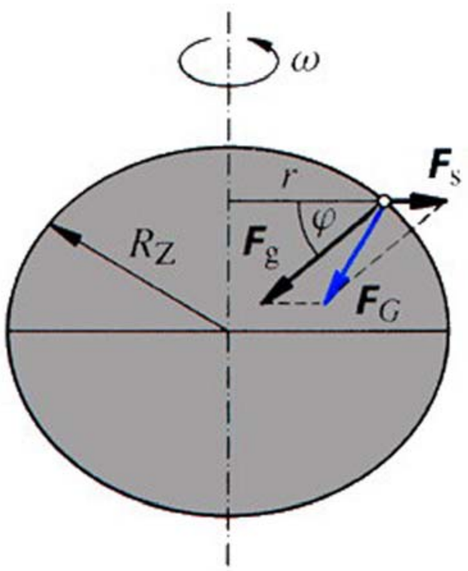
2. 1. Gravitational and gravity force

Now we know that around every body is a gravitational field, which shows itself by power action at other bodies. From Newton's law of gravitation emerges that the value of the power action depends on bodies' sizes and also their distance.

Gravitational force awards the body of weight *m* in given place gravitational acceleration.

$$\mathbf{a}_g = \frac{\mathbf{F}_g}{\mathbf{m}}$$

This means: intensity of the gravitational field in given place equals gravitational acceleration, which gravitational force awards the body in this place ($\boldsymbol{K} = \boldsymbol{A}_g$). On the Earth's surface is gravitational acceleration $\boldsymbol{a}_g$ = 9,83 m.s^{-2}.



We "use" gravity force $\boldsymbol{F}_G$ and gravity acceleration **g** on Earth, but these are not the same as gravitational force or gravitational acceleration. That is because the Earth rotates on its own axis. Also the inertial centrifugal force of Earth's rotating on its own axis (rotating system is inertia less system) $\boldsymbol{F}_s$ takes place on the Earth's surface, so the total gravity force $\boldsymbol{F}_G$ equals their vector sum. Its direction is considered as vertical and we determine it with a free-hanging plumb line.

If we pursue gravitational force's affects in small areas of the Earth's gravitational field, we can consider gravitational acceleration and gravitational force as constant, because the changes of their affect's value and direction approach zero.

2. 2. Gravity force and body's weight

Vertical direction is direction of gravity force and gravity direction, but it is not always the direction to the Earth's centre (gravity force heads for the Earth's centre only on the geographical poles and on the equator). Area, where the gravity force affects itself, is standing for a gravity field.

This is a formula for the centrifugal force's $\boldsymbol{F_s}$ value:

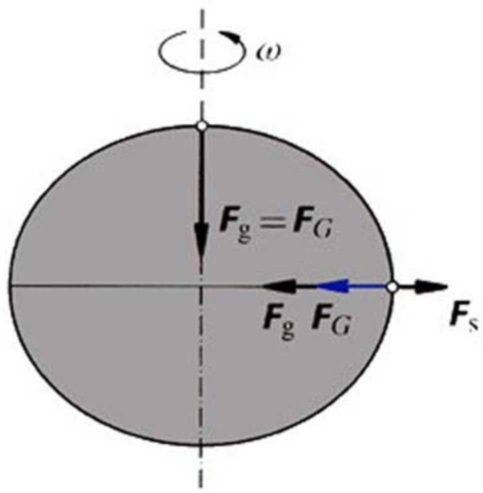
$$F_S = \frac{m \cdot v^2}{R_Z} = \frac{m \cdot \omega^2 \cdot r^2}{R_Z} = m \cdot \omega^2 \cdot R_Z \cdot \cos^2 \alpha \qquad \omega = \frac{2 \cdot \pi}{T}$$

$$T = 1 \text{ day}$$

$$E = m \cdot g \cdot h$$

r is the distance from a place on the Earth's surface from the axis of rotating, ω is the angelical speed of Earth's rotating, R_Z is the Earth's radius, α is the latitude of the place. This means that the biggest centrifugal force is on the equator and that it equals zero on the poles. Value of the gravity acceleration depends on the latitude and also on the altitude. It is 9,78 $m.s^{-2}$ on the equator at the sea level and 9,83 $m.s^{-2}$ on the geographical poles. It is 9,81 $m.s^{-2}$ in central Europe; the normal gravity acceleration was created by an arrangement and its value is 9,80665 $m.s^{-2}$ (at the sea level on the 45° of north latitude).

The gravity acceleration could be considered as homogeneous in a small area of the Earth's surface. We distinguish the body's weight ***G*** from the gravity force magnitude $\boldsymbol{F_G}$. Gravity force rises by the instrumentality of gravity field on a body. Body's weight means incidence of a body placed in the Earth's gravitational field on other bodies. Gravity force and weight have also another place of work. Gravity force's place of work is the body's centre of gravity; weight's place of work is on the body's interface area with a mat or in the point of hanging. Both of these two forces have the same value and direction on the given place of the Earth's surface. The difference between them is the clearest in a weightless state, for example at free fall. If we hang a body at a dynamometer in a rest position, the body's weight will stretch the spring and the dynamometer will measure the body's weight. If we drop the body with the dynamometer, the body will stop stretching the string and the dynamometer will measure zero, but the body (with the dynamometer) continues in a flight, because the gravity force is affecting it.



3. Body's movements in homogenous gravitational field

Throws are bodies' movements, which take place near the Earth and which trajectories could be vanished with regard to the Earth's proportions. We will also suppose that there is neither any other force affecting the bodies than the gravity force nor the air resistance. So we suppose that the movements take place in vacuum.

We can observe free fall and compound movements (throws) in the Earth's homogenous gravity field. These movements are composed of free fall and uniform rectilinear movement. We divide them by the movement's direction:

1. Vertical throw downwards
2. Vertical throw upwards
3. Horizontal throw
4. Oblique throw upwards

3. 1. Free fall

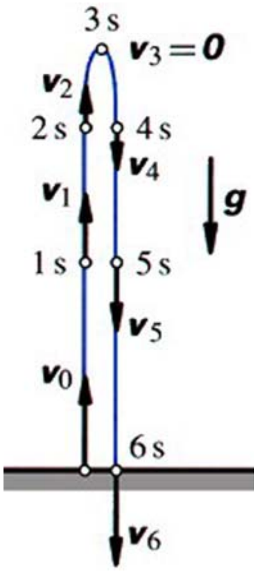
The easiest movement in the Earth's gravity field is the free fall. The free fall is uniformly accelerated movement with gravity acceleration *g* and with zero initial velocity. The free fall is a part of all the complicated movements in the Earth's homogenous gravity field.

These formulas are for the instantaneous velocity and trajectory of the free fall:

$$v = g \cdot t \qquad s = \frac{g \cdot t^2}{2}$$

3. 2. Vertical throw upwards

Vertical throw upwards is composed of free fall and uniform rectilinear movement upwards. For example, the ball, which a tennis player throws before service, a stone blowing up from a volcano etc. These formulas hold for the instantaneous velocity *v*, travel *s* in time *t*:



$$v = v_0 - g \cdot t$$
$$s = v_0 \cdot t - \tfrac{1}{2} \cdot g \cdot t^2$$

v_0 je počáteční rychlost

The part of the vertical throw upwards when a mass point rises is called climb; the mass point performs uniformly slowed-down movement during it. The climb finishes when the instantaneous velocity equals zero, after it starts the free fall. The time when the mass point rises is the time of the climb *T*, and the mass point rises in the height *H*.

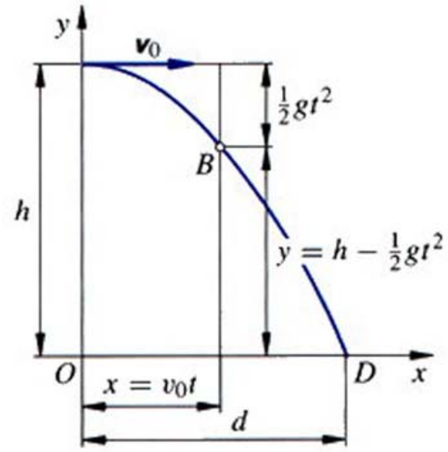
The trajectory of vertical throw upwards is in fact a line. The climb and the freefall are separated for clearness in the picture.

$$H = v_0 \cdot T - \frac{1}{2} \cdot g \cdot T^2$$
$$H = v_0 \cdot \frac{v_0}{g} - \frac{1}{2} \cdot g \cdot \frac{v_0^2}{g^2}$$
$$H = \frac{v_0^2}{2 \cdot g}$$

Vertical throw downwards is composed of free fall and uniform rectilinear movement downwards. It is uniformly accelerated movement with ***g*** acceleration and initial velocity $\boldsymbol{v_0}$. These movements happen when we throw a stone into a chasm. The difference between this and a free fall is that we only let the stone fall down from a rest position by the free fall (v_0 = 0 m.s^{-1}).

3. 3. Horizontal throw

Horizontal throw is composed of free fall and uniform rectilinear movement, whose direction is horizontal with the Earth's surface. The trajectory is a part of a parabola with apex in the place of the throw's start. Examples: effluent liquid from horizontally held hose, a marble, which goes over a horizontal table's edge.



The throw's length depends on the initial velocity $\boldsymbol{v_0}$ and on the height *H*, from which the body was thrown. We have to divide the movement in two parts – horizontal and vertical – to find out the mass point position.

The vertical movement is a free fall from the *H* height and the horizontal movement is a uniform rectilinear movement. We can determine the instantaneous position and velocity as a sum of these two movements, the instantaneous height *h* and the distance from the point of landing *d*.

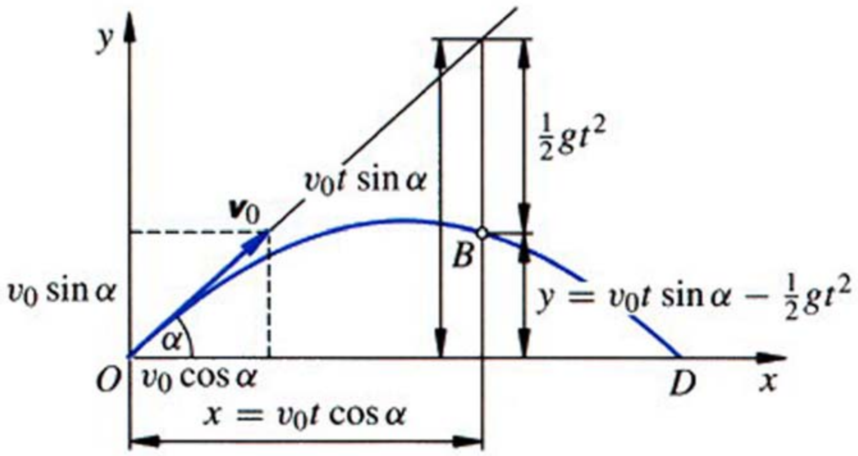
$$d = v_0 \cdot t \qquad h = H - \frac{g \cdot t^2}{2} \qquad T = \sqrt{\frac{2 \cdot H}{g}} \qquad D = v_0 \cdot \sqrt{\frac{2 \cdot H}{g}}$$

We can also determine the instantaneous velocity of the horizontal throw as a vector sum of the vertical and the horizontal velocity, where the horizontal velocity is the initial velocity and vertical one conforms to the free fall.

3. 4. Oblique throw upwards

The oblique throw upwards consists of free fall and uniform rectilinear movement slope to the Earth's surface. The throw's length depends on the initial speed $\boldsymbol{v_0}$ and on the elevation angle α.

If we want to determine the mass point's position and velocity during the oblique throw, we have to disarticulate it in vertical and horizontal movement. We also disarticulate the initial velocity in horizontal initial velocity $\boldsymbol{v_x}$ and vertical initial velocity $\boldsymbol{v_y}$.



The instantaneous velocity is a vector sum of vertical and horizontal velocity. The instantaneous velocity could be determined as by the vertical throw upwards and the horizontal velocity is the same all the time.

$$v_X = v_0 \cdot \cos\alpha$$
$$v_Y = v_0 \cdot \sin\alpha$$
$$x = v_X \cdot t = v_0 \cdot t \cdot \cos\alpha$$
$$y = v_Y \cdot t - \frac{g \cdot t^2}{2} = v_0 \cdot t \cdot \sin\alpha - \frac{g \cdot t^2}{2}$$

3. 5. Security parabola

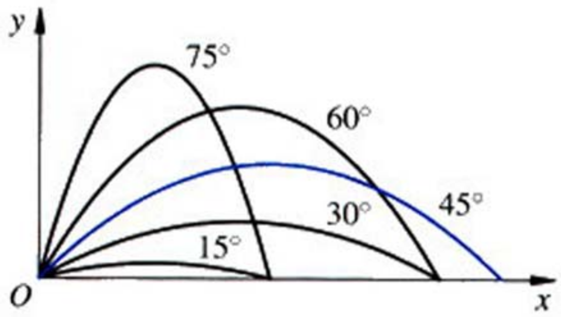
Security parabola is a curve, which defines an area, where it is possible to hit a given point (with constant velocity). If you are out of the parabola, you can not be hit. This pattern is important for artillerymen (they are interested in the shot on the *x* axis), because it border places, which they can hit.

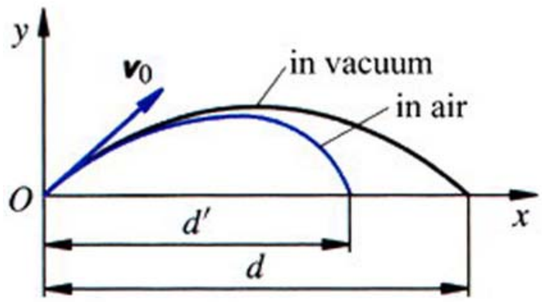
If you want to see the security parabola, click on the simulation button in the 'Show' frame.

3. 6. Oblique throw upwards with indispensable air resistance

The important value of the oblique throw is the throw's length. In military terminology it is called a shot. The throw's length is the biggest for the elevation angle of 45°. It is the same for pairs *α* and (90 – *α*) … 15° and 75°; 30° and 60°. For example, a shot from a cannon (*α* < 45°), from a mine-thrower (*α* > 45°).

The trajectory of the oblique throw in a vacuum is a parabola and a ballistic curve in the air. A ballistic curve is always shorter than the parabola, because the air resistance is also affecting the mass point (works against the horizontal part of the velocity).





$$D = v_x \cdot T_D = v_0 \cdot T_D \cdot \cos\alpha$$

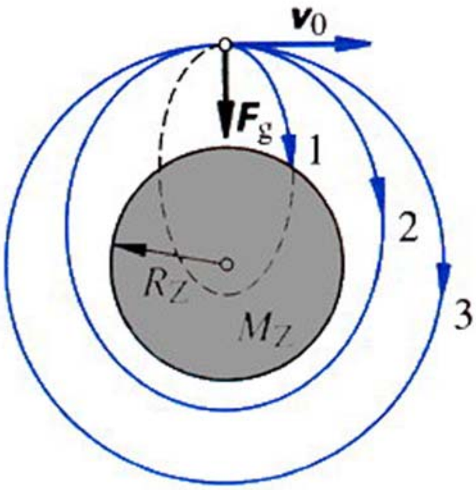
$$T_D = \frac{2 \cdot v_y}{g} = \frac{2 \cdot v_0 \cdot \sin\alpha}{g}$$

$$D = \frac{2 \cdot v_0^2 \cdot \sin\alpha \cdot \cos\alpha}{g} = \frac{v_0^2 \cdot \sin(2 \cdot \alpha)}{g}$$

4. Body's movements in central Earth's gravitational field

Throws are bodies' movements in a homogenous gravity field. We have to consider the gravity field as central for the movements of rockets, satellites or spaceships. The satellites' trajectories depend on its velocity.

4. 1. Circular rate



1) Quite small initial velocity – the body is moving on an ellipse before it hit the Earth's surface. The part of the ellipse rises with the initial velocity.

2) The body does not hit the Earth in bigger initial velocities, but it circumscribes the whole ellipse.

3) If the initial velocity is a circular velocity $\boldsymbol{v_k}$, the body circumscribes a circle with a centre in the Earth's centre. The Earth's gravitational force $\boldsymbol{F_g}$ and the centrifugal force $\boldsymbol{F_0}$ are in balance.

$$F_g = F_o$$

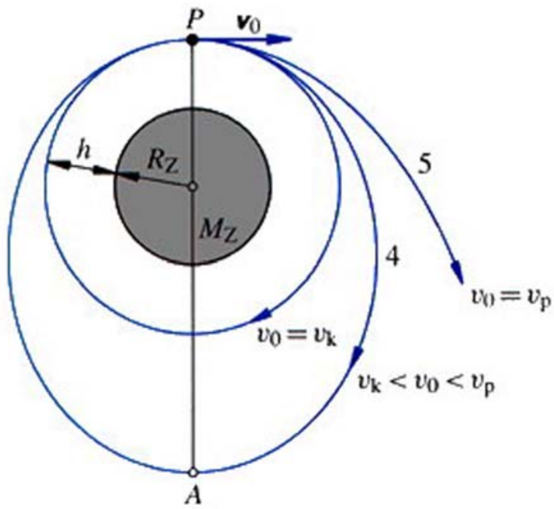
$$\kappa \cdot \frac{m \cdot M_Z}{(R_Z + h)^2} = \frac{m \cdot v_K^2}{R_Z + h}$$

$$v_K = \sqrt{\frac{\kappa \cdot M_Z}{R_Z + h}}$$

The value of the circular velocity v_k near the Earth's surface is 7,9 km·s^{-1}.

4. 2. Escape velocity

4) The trajectory is elliptical in bigger initial velocities again. The ellipse's plane goes through the Earth's centre, where it is its focus. The **P** point, where the distance from the Earth is the smallest, is called *perigee*, the **A** point, where the distance from the Earth is the biggest, is called *apogee*. With rising velocity the ellipse is more oblong.



5) The trajectory changes in a parabola in initial velocity and the body recedes from the Earth. The velocity $\boldsymbol{v_p}$ is called parabolic, escape. For the mentioned $\boldsymbol{v_k}$ = 7,9 km.s^{-1} is $\boldsymbol{v_p}$ = 11,2 km.s^{-1}.

$$v_p = v_k \cdot \sqrt{2}$$

6) Before the body reaches hyperbolic speed, it is moving in the Sun's gravitational field. When it reaches this velocity it leaves the Solar system.

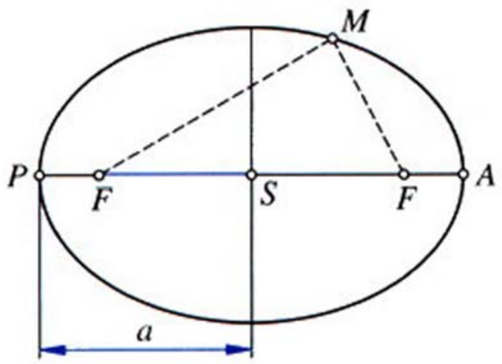
5. Body's movements in central Sun's gravitational field

Now you will get to know the basic patterns, which affect the bodies' movements in a Solar system (planets, asteroids etc.). They are described by Kepler's laws. And there is a physical problem for you of course, so you will test yourself if you had understood everything.

5. 1. First Kepler's law

The planets orbits around the Sun on ellipses are only a bit different from circles. Its shared focus is the Sun.

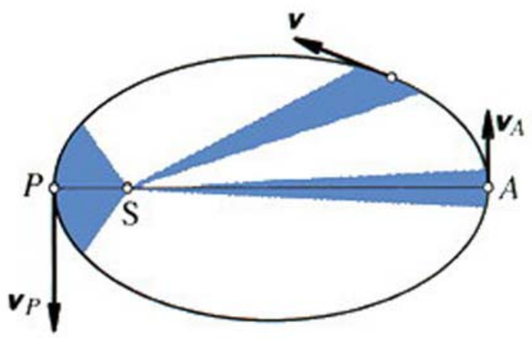
The apex **P**, where the planet is nearest to the Sun, is called *perihelion*. The apex **A**, where the planet is furthest from the Sun, is called *aphelion*.



5. 2. Second Kepler's law

Planes, which are circumscribed by the planet's circulant in a unit of time, are constant.

The circulant is an abscissa, which connects the planet's centre with the Sun's centre. The consequence of this law is that the planets are moving faster in the *perihelion* and slower in the *aphelion*.



5. 3. Third Kepler's law

The quotient of square of the planets' orbiting time equals the cube of their main half-axles.

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

If we consider that the planets are moving on ellipses only a bit different from circles, the average distance from the Sun could be used instead of the main half-axles.

Kepler's laws do not work only in our Solar system, but also for bodies orbiting around the Earth (Moon, satellites, etc.)

The distances in the Solar system are measured in astronomical units AU, which equal the average distance between the Sun and the Earth.

5. 4. Solar system

Nine planets (or maybe ten, the planet's status for Sedna has not been accepted yet), enormous amount of asteroids, planetoids and some comets, which visit the Solar system once for hundred years and leave again, are moving in our Solar system. Moons, bands of dust and in the Earth's case also enormous amount of satellites, remains of space vehicles or other space litter orbit around the planets. Kepler's laws hold either for Jupiter or for a screw from MIR. Do you think that it is very complicated? Wait for a while, because these bodies affect each other, they gravitate and hit each other and this takes part for ages. If a planet, a moon or a sun do not attract them to it, they will finish as a crater or as gas in their atmosphere.

6. Summary

You should now understand a bit of the laws controlling the bodies' movements in the gravitational field. You can not do the astronomical calculation, because our formulas do not include mutual interaction of the gravitational fields nor complicated or long-term effects.

If you do not trust us for some reason, do this experiment: Take the Moon and let it orbit between the Earth and Mars and now observe it at least for ten thousand years. Work out where it should be and where it is ... we were right, weren't we?

Now seriously, the described laws are difficult to prove, because they do not include wind, air resistance, friction (movements in homogenous gravity field near the Earth's surface) etc. If you want to try some laws, take a metal ball or a stone and let it fall from a higher building (be careful of the passers-by), measure the building's height, the ball's weight and the time of falling, work out the length of the fall. Is it true? This experiment has one big advantage: you would need a tornado or a very high building (the air resistance will be noticeable) to influence it. If you have some questions to these problems ask your physic's teacher. He or she will be glad to give you the interpretation.

We hope you enjoyed our project and thanks for your attention.